Write your name here

Surname	Other names

Edexcel IGCSE

Centre Number	Candidate Number

Mathematics A

Paper 3H

Higher Tier

Monday 6 June 2011 – Afternoon
Time: 2 hours

Paper Reference
4MA0/3H

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P38579A

6/6/6/6/6/4/3

P 3 8 5 7 9 A 0 1 2 4

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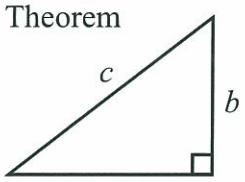
IGCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

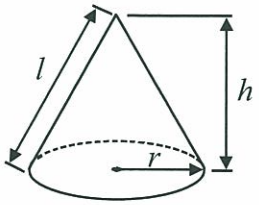
Pythagoras' Theorem



$a^2 + b^2 = c^2$

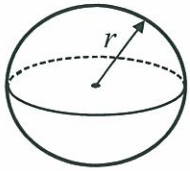
Volume of cone = $\frac{1}{3}\pi r^2 h$

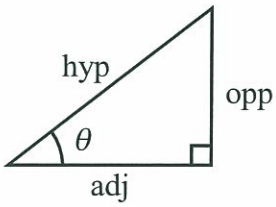
Curved surface area of cone = $\pi r l$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$





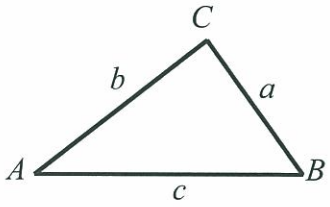
adj = hyp × cos θ
opp = hyp × sin θ
opp = adj × tan θ

or $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

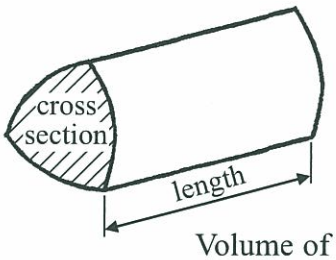
In any triangle ABC



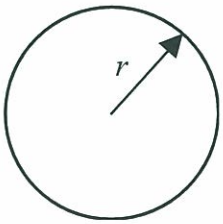
Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$



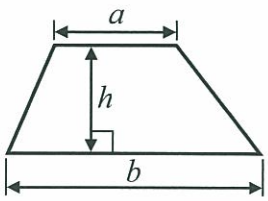
Volume of prism = area of cross section × length

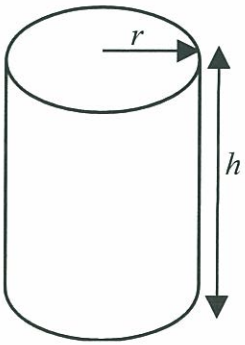


Circumference of circle = $2\pi r$

Area of circle = πr^2

Area of a trapezium = $\frac{1}{2}(a + b)h$





Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi rh$

The Quadratic Equation
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Use your calculator to work out the value of

$$\frac{24.1}{8.4-7.8} - 6.2^2$$

Write down all the figures on your calculator display.

1.726666667

(2)

(b) Give your answer to part (a) correct to 3 significant figures.

1.73

(1)

(Total for Question 1 is 3 marks)

2

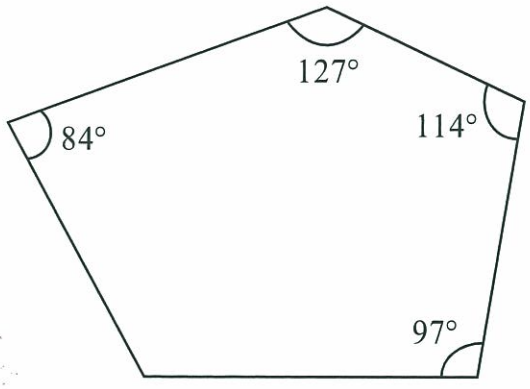


Diagram **NOT** accurately drawn

Four of the angles of a pentagon are 97°, 114°, 127° and 84°.

Work out the size of the fifth angle.

Sum of interior angles of an n-sided polygon is given by $180(n-2)$.

$\Rightarrow$ Sum of interior angles of a 5-sided polygon (i.e. a pentagon) is given by $180(5-2) = 180(3) = 540°$.

$540 - (84 + 127 + 114 + 97)$

$= 540 - 422 = 118°$

118 °

(Total for Question 2 is 4 marks)

3 (a) Factorise $w^2 - 9w$.

$w(w-9)$

(2)

(b) Solve $5x - 1 = 2x - 7$

$3x - 1 = -7$

$3x = -6$

$\Rightarrow x = -\frac{6}{3} = -2$

$x = -2$

(3)

(c) Expand and simplify $(y - 7)(y + 3)$.

$y^2 + 3y - 7y - 21$

$= y^2 - 4y - 21$

$y^2 - 4y - 21$

(2)

(Total for Question 3 is 7 marks)

4 Every morning, Samath has one glass of fruit juice with his breakfast.
He chooses at random orange juice or pineapple juice or mango juice.
The probability that he chooses orange juice is 0.6
The probability that he chooses pineapple juice is 0.3

(a) Work out the probability that he chooses mango juice.

$P(\text{Mango juice}) = 1 - 0.6 - 0.3 = 0.1$

0.1

(2)

(b) There are 30 days in April.

Work out an estimate for the number of days in April on which Samath chooses orange juice.

$30 \times P(\text{orange juice}) = 30 \times 0.6 = 18 \text{ days}$

18

(2)

(Total for Question 4 is 4 marks)

---

5 Show that $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$

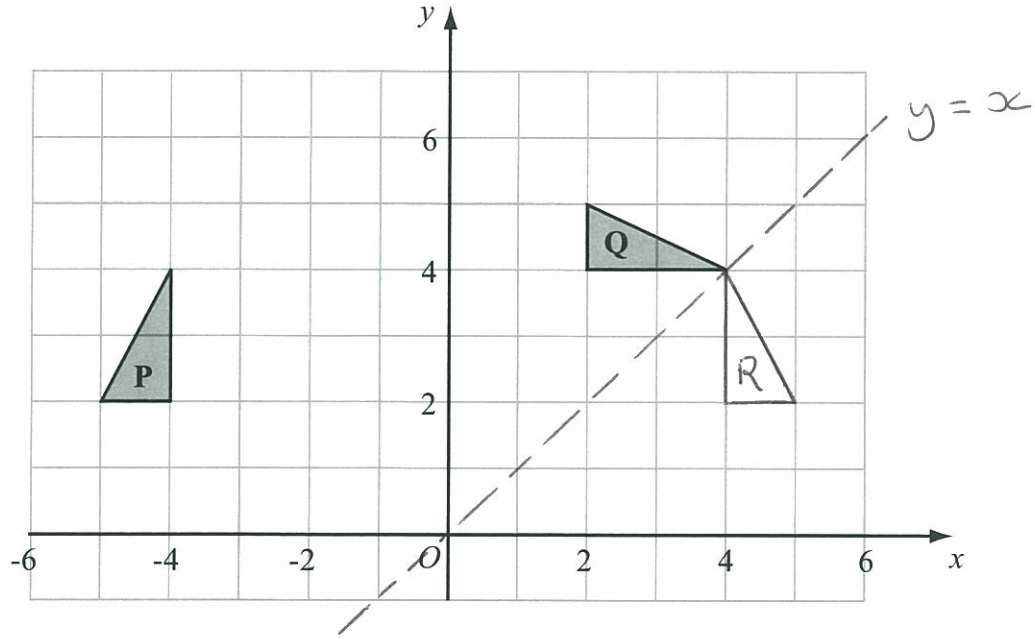
$\frac{5}{6} - \frac{3}{4} = \frac{2}{2}\left(\frac{5}{6}\right) - \frac{3}{3}\left(\frac{3}{4}\right) = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

ALTERNATIVELY, USE CAMPING (CROSS-MULTIPLYING) METHOD TO EVALUATE $\frac{5}{6} - \frac{3}{4}$.

$\frac{5}{6} - \frac{3}{4} = \frac{5(4) - 3(6)}{6 \times 4} = \frac{20 - 18}{24} = \frac{2}{24} = \frac{1}{12}$

(Total for Question 5 is 2 marks)

6



(a) Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

A rotation 90° clockwise about the point (0, 0)

(3)

(b) Reflect triangle **Q** in the line $y = x$.

Label the new triangle **R**.

(2)

(Total for Question 6 is 5 marks)

7 The perimeter of a triangle is 90 cm.
The lengths of the sides of the triangle are in the ratios 3 : 5 : 7

Work out the length of the longest side of the triangle.

$\frac{90}{3+5+7} \times 7 = \frac{90}{15} \times 7 = 6 \times 7 = 42$ cm

42 cm

(Total for Question 7 is 3 marks)

8 $\mathscr{E}$ = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
A = {odd numbers}
P = {prime numbers}

List the members of the set

(i) $A \cap P$,

3, 5, 7, 11

(ii) $A \cup P$.

2, 3, 5, 7, 9, 11

(Total for Question 8 is 2 marks)

9 Ella invested \$8000 for 3 years at 5% per annum **compound interest**.

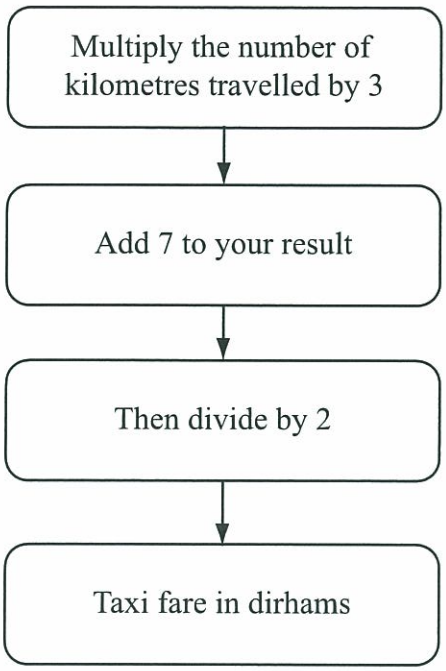
Calculate the value of her investment at the end of 3 years.

$8000 \times 1.05^3 = \$9,261$

\$ 9,261

(Total for Question 9 is 3 marks)

10 This rule can be used to work out the fare, in dirhams, for a taxi journey in Dubai.



Find a formula for the fare, C dirhams, for a taxi journey of d kilometres.

$C = \frac{3d + 7}{2}$ or $\frac{1}{2}(3d + 7)$

$C = \frac{3d + 7}{2}$

(Total for Question 10 is 3 marks)

11 The table shows information about the weights of 80 parcels.

Weight (w kg)	Frequency
$0 < w \leqslant 2$	8
$2 < w \leqslant 4$	14
$4 < w \leqslant 6$	26
$6 < w \leqslant 8$	17
$8 < w \leqslant 10$	10
$10 < w \leqslant 12$	5

(a) Work out an estimate for the total weight of the 80 parcels.

$8(1) + 14(3) + 26(5) + 17(7) + 10(9) + 5(11)$

$= 444$ kg

444 kg

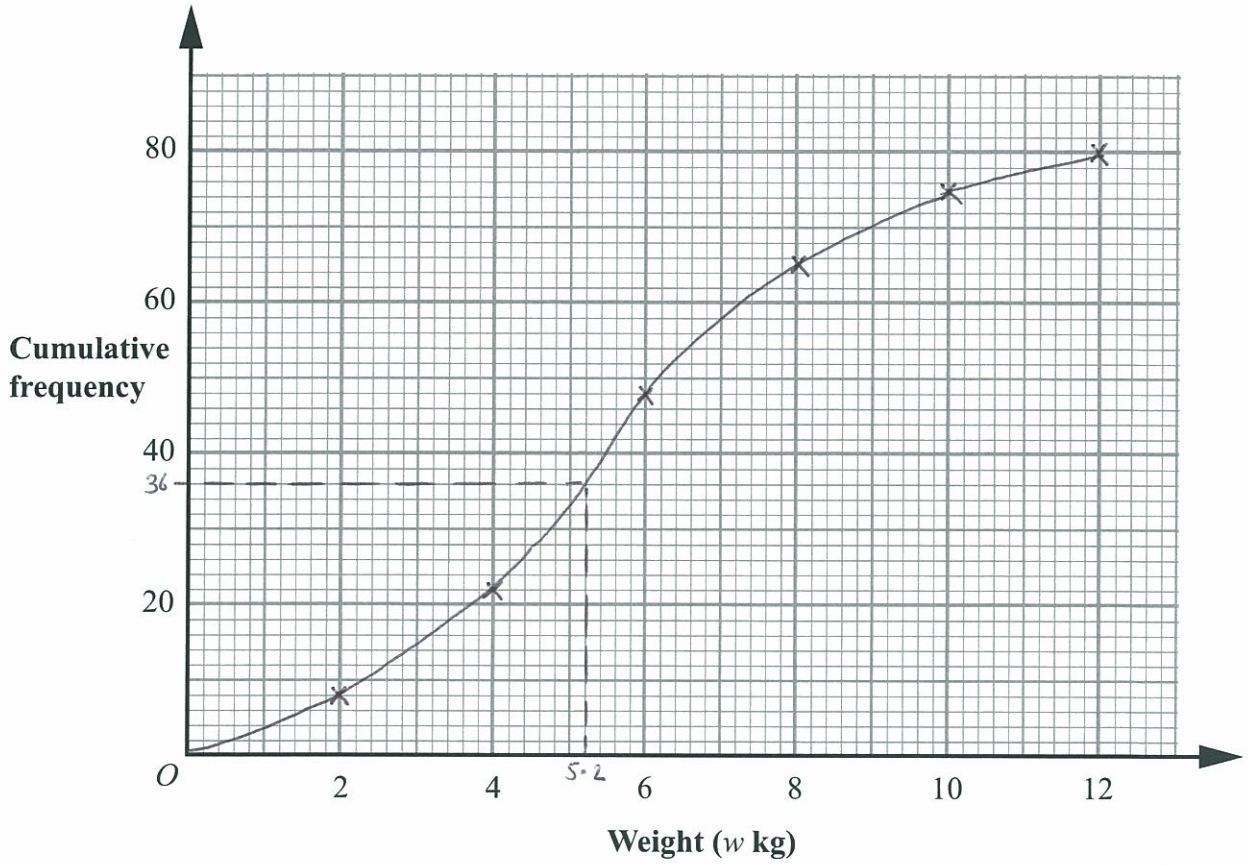
(3)

(b) Complete the cumulative frequency table.

Weight (w kg)	Cumulative frequency
$0 < w \leqslant 2$	8
$0 < w \leqslant 4$	22
$0 < w \leqslant 6$	48
$0 < w \leqslant 8$	65
$0 < w \leqslant 10$	75
$0 < w \leqslant 12$	80

(1)

(c) On the grid, draw a cumulative frequency graph for your table.



(2)

(d) Use the graph to find an estimate for the number of parcels which weighed less than 5.2 kg.

36

(2)

(Total for Question 11 is 8 marks)

12

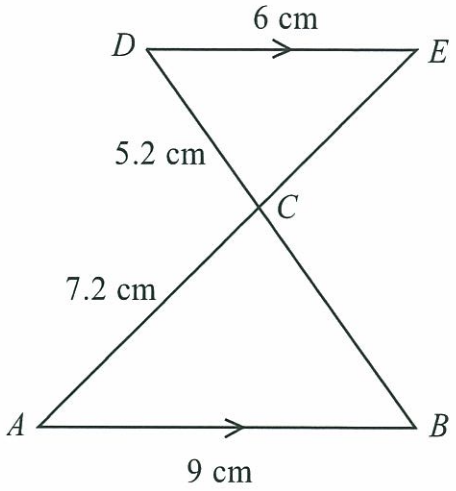


Diagram **NOT** accurately drawn

AB is parallel to *DE*.
ACE and *BCD* are straight lines.
AB = 9 cm.
AC = 7.2 cm.
CD = 5.2 cm.
DE = 6 cm.

(a) Calculate the length of *BC*.

Triangles ABC and CDE are similar (all angles match) with side BC corresponding to CD and AC corresponding to CE.

$\Rightarrow \frac{AC}{CE} = \frac{BC}{CD} = \frac{AB}{DE}$

$\frac{BC}{5.2} = \frac{9}{6}$

$\Rightarrow BC = \frac{3}{2} \times 5.2 = 7.8$ cm

7.8 cm

(2)

(b) Calculate the length of *CE*.

$\frac{AC}{CE} = \frac{AB}{DE}$

$\Rightarrow \frac{7.2}{CE} = \frac{3}{2}$

$\Rightarrow CE = 7.2\left(\frac{2}{3}\right) = 4.8$ cm

4.8 cm

(2)

(Total for Question 12 is 4 marks)

13 Solve $\frac{2x-1}{4}+\frac{x-1}{5}=2$

$$\frac{5(2x-1)+4(x-1)}{20}=2$$

$$10x-5+4x-4=40$$

$$14x-9=40$$

$$\Rightarrow x=\frac{40+9}{14}=\frac{49}{14}=\frac{7}{2}=3.5$$

$x =$ 3.5

(Total for Question 13 is 4 marks)

14 $y = 1.8$ correct to 1 decimal place.

Calculate the lower bound for the value of $4y + 1$

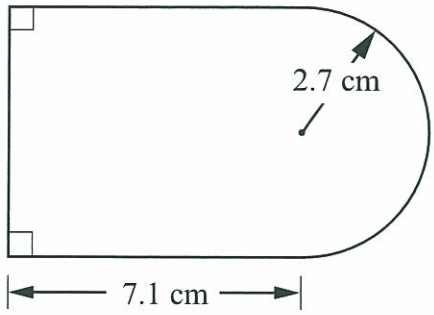
$$4(1.75)+1=8$$

8

(Total for Question 14 is 2 marks)

15 (a) Here is a shape made from a rectangle and a semicircle.

Diagram **NOT** accurately drawn



The length of the rectangle is 7.1 cm.
The radius of the semicircle is 2.7 cm.

Work out the area of the shape.
Give your answer correct to 3 significant figures.

N.B: Area of a circle = πr^2

$\Rightarrow$ Area of a semi-circle = $\frac{1}{2}\pi r^2$

Area of shape = $(7.1 \times 5.4) + \frac{1}{2}\pi(2.7^2)$

$= 49.8\ cm^2$ (3s.f.).

49.8 cm^2

(4)

(b) Here is another shape made from a rectangle and a semicircle.

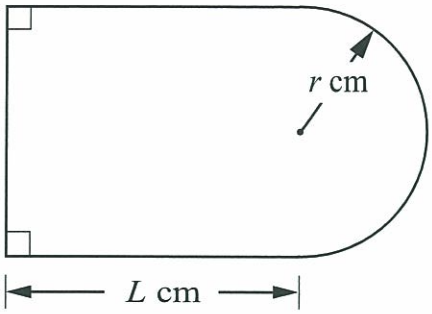


Diagram **NOT** accurately drawn

The length of the rectangle is L cm.
The radius of the semicircle is r cm.

The perimeter, P cm, of the shape is given by the formula

$$P = \pi r + 2L + 2r$$

Make r the subject of the formula $P = \pi r + 2L + 2r$.

$P - 2L = \pi r + 2r$

$P - 2L = r(\pi + 2)$

$\Rightarrow r = \dfrac{P - 2L}{\pi + 2}$

$r = \dfrac{P - 2L}{\pi + 2}$

(3)

(Total for Question 15 is 7 marks)

16

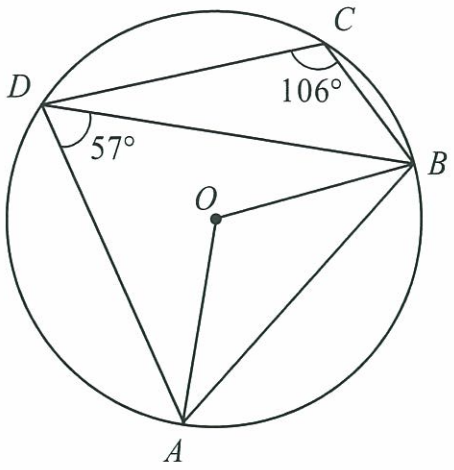


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*.
Angle *ADB* = 57°.
Angle *BCD* = 106°.

(a) (i) Calculate the size of angle *AOB*.

$A\hat{O}B = 2(57) = 114°$

114 °

(ii) Give a reason for your answer.

The angle subtended by an arc at the centre is double the angle subtended by the same arc at a point on the circumference

(2)

(b) Calculate the size of angle *BAD*.

$B\hat{A}D = 180 - 106 = 74°$

(Opposite angles of a cyclic quadrilateral add to 180°).

74 °

(1)

(Total for Question 16 is 3 marks)

17 Here are seven counters.
Each counter has a number on it.

Ali puts the seven counters in a bag.
He takes, at random, a counter from the bag and does **not** replace the counter.
He then takes, at random, a second counter from the bag.

Calculate the probability that

(i) the number on the second counter is 2 more than the number on the first counter,

P(2nd counter being 2 more than 1st) = P(2 and then 4)

$= \frac{1}{7} \times \frac{2}{6} = \frac{1}{7} \times \frac{1}{3} = \frac{1}{21}$

$\frac{1}{21}$

(ii) the number on the second counter is 1 more than the number on the first counter.

P(1 and 2 OR 4 and 5)

= P(1 and 2) + P(4 and 5)

$= \frac{1}{7}\left(\frac{1}{6}\right) + \frac{2}{7}\left(\frac{3}{6}\right)$

$= \frac{1}{42} + \frac{6}{42} = \frac{7}{42} = \frac{1}{6}$

$\frac{1}{6}$

(Total for Question 17 is 5 marks)

18

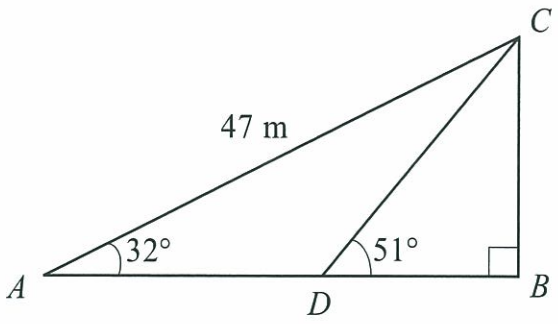


Diagram **NOT** accurately drawn

Triangle ABC is right-angled at B.
Angle $BAC = 32°$
$AC = 47$ m.
D is the point on AB such that angle $BDC = 51°$

Calculate the length of BD.
Give your answer correct to 3 significant figures.

$BD \times \tan 51° = BC$

and $BC = 47(\sin 32°)$

$\Rightarrow BD \times \tan 51° = 47(\sin 32°)$

$\therefore BD = \dfrac{47 \sin 32°}{\tan 51°} = 20.2$ m (3 s.f.).

20.2 m

(Total for Question 18 is 5 marks)

19 P is directly proportional to the cube of Q.
When $Q = 15$, $P = 1350$

(a) Find a formula for P in terms of Q.

$P \propto Q^3$

$\Rightarrow P = kQ^3$

$\Rightarrow 1350 = 15^3 k$

$\Rightarrow k = \frac{1350}{15^3} = 0.4 \quad \therefore P = 0.4Q^3$

$P = 0.4Q^3$

(3)

(b) Calculate the value of P when $Q = 20$

$P = 0.4(20^3) = 3200$

$P = 3200$

(1)

(Total for Question 19 is 4 marks)

---

20 $x = a \times 10^n$ where n is an integer and $\sqrt{10} \leqslant a < 10$

Find, in standard form, an expression for x^2.
Give your expression as simply as possible.

$x^2 = (a \times 10^n)^2$

$x^2 = a^2 \times 10^{2n}$

If $\sqrt{10} \leqslant a < 10$, then $10 \leqslant a^2 < 100$

So $x^2 = \frac{a^2}{10} \times 10^{2n} \times 10$

$= 0.1a^2 \times 10^{2n+1}$

$x^2 = 0.1a^2 \times 10^{2n+1}$

(Total for Question 20 is 3 marks)

21

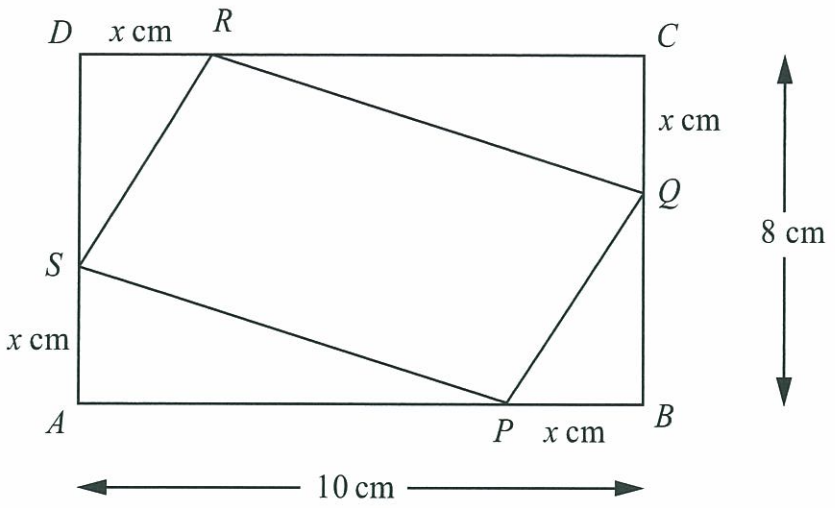


Diagram **NOT** accurately drawn

ABCD is a rectangle.
AB = 10 cm.
BC = 8 cm.
P, *Q*, *R* and *S* are points on the sides of the rectangle.
BP = *CQ* = *DR* = *AS* = *x* cm.

(a) Show that the area, *A* cm², of the quadrilateral *PQRS* is given by the formula

$$A = 2x^2 - 18x + 80$$

$A = 10(8) - x(10 - x) - x(8 - x)$

$= 80 - 10x + x^2 - 8x + x^2$

$= 2x^2 - 18x + 80$

(3)

(b) For $A = 2x^2 - 18x + 80$

(i) find $\frac{dA}{dx}$,

$4x - 18$

(ii) find the value of x for which A is a minimum.

$\frac{dA}{dx} = 0 \Rightarrow 4x - 18 = 0$

$\Rightarrow x = \frac{0+18}{4} = 4.5$

$x = 4.5$

(iii) Explain how you know that A is a minimum for this value of x.

One way is to observe that the coefficient of the x^2 term in the original quadratic function for A is positive (i.e. +2). and so our function forms a trough instead of a crest. * (5)

(Total for Question 21 is 8 marks)

* For any quadratic equation of the form $y = ax^2 + bx + c$, the associated curve forms a trough if $a > 0$ ∪

and forms a crest if $a < 0$ ∩

22 Solve the simultaneous equations

$$y = 2x - 3$$

$$x^2 + y^2 = 2$$

Replace y with $2x - 3$ in the second equation and solve for x:

$$x^2 + (2x - 3)^2 = 2$$

$$x^2 + 4x^2 - 12x + 9 - 2 = 0$$

$$5x^2 - 12x + 7 = 0$$

$$(5x - 7)(x - 1) = 0$$

$$\Rightarrow x = \frac{7}{5} \text{ or } 1$$

From equation ①,

When $x = \frac{7}{5}$, $y = 2\left(\frac{7}{5}\right) - 3 = -\frac{1}{5}$

and when $x = 1$, $y = 2(1) - 3 = -1$

$(x, y) = (1.4, -0.2)$ or $(1, -1)$

(Total for Question 22 is 6 marks)

23

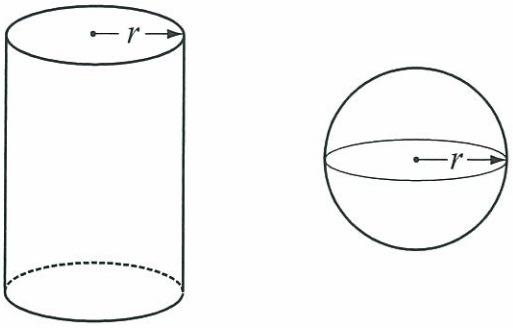


Diagram **NOT** accurately drawn

Let h = height of cylinder

The diagram shows a solid cylinder and a solid sphere.
The cylinder has radius r.
The sphere has radius r.

Given that $\dfrac{\textbf{Total} \text{ surface area of cylinder}}{\text{Surface area of sphere}} = 2$ $\longrightarrow \dfrac{2\pi r^2 + 2\pi rh}{4\pi r^2} = 2$

find the value of $\dfrac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

$\Rightarrow 2\pi r^2 + 2\pi rh = 8\pi r^2$

$\Rightarrow h = \dfrac{6\pi r^2}{2\pi r} = 3r$

$\dfrac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \dfrac{\pi r^2 h}{\frac{4}{3}\pi r^3} = \dfrac{\pi r^2 (3r)}{\frac{4}{3}\pi r^3}$

$= \dfrac{3\pi r^3}{\frac{4}{3}\pi r^3} = \dfrac{3}{(4/3)} = \dfrac{9}{4} = 2\frac{1}{4}$ or 2.25.

N.B: $\dfrac{3}{\left(\frac{4}{3}\right)} \equiv 3 \div \frac{4}{3} = 3 \times \frac{3}{4} = \frac{9}{4} = 2.25$

2.25

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

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