Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
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14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
TOTAL	

General Certificate of Secondary Education
Higher Tier
March 2013

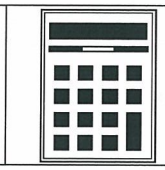
Mathematics 43603H

Unit 3

Wednesday 6 March 2013 9.00 am to 10.30 am

H

For this paper you must have:
- a calculator
- mathematical instruments.



Time allowed
- 1 hour 30 minutes

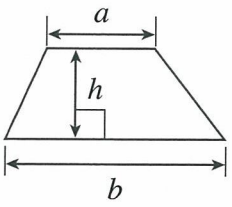
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book.
- If your calculator does not have a π button, take the value of π to be 3.14 unless another value is given in the question.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- The quality of your written communication is specifically assessed in Questions 6 and 15. These questions are indicated with an asterisk (*).
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

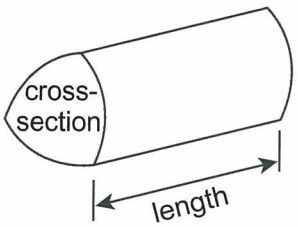
Advice
- In all calculations, show clearly how you work out your answer.

Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a+b)h$

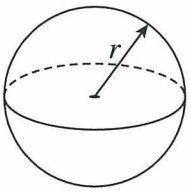


Volume of prism = area of cross-section × length



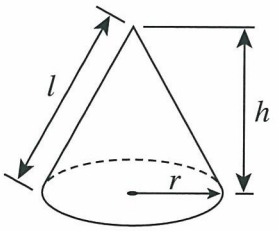
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

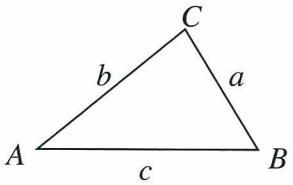
Curved surface area of cone = $\pi r l$



In any triangle ABC

Area of triangle = $\frac{1}{2}ab\sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

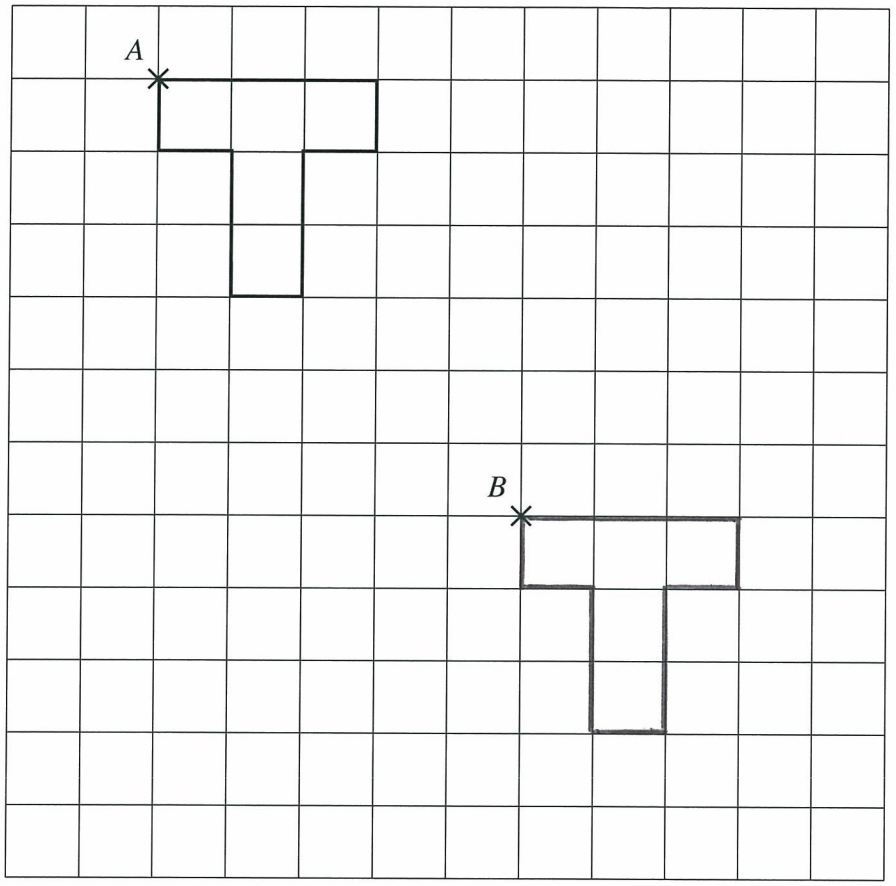
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Answer **all** questions in the spaces provided.

1 (a) Translate this T-shape so that point A moves to point B.



(1 mark)

1 (b) Describe the translation.

A translation by the vector $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

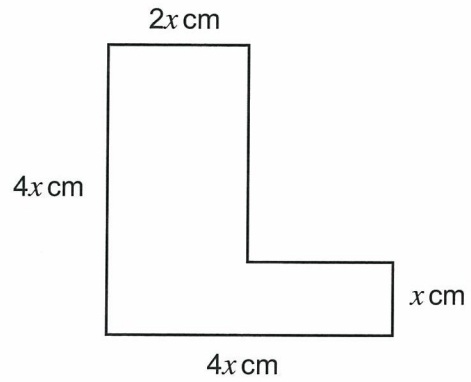
(2 marks)

3

Turn over ▶

2 The perimeter of this L-shape is 56 cm.

Not drawn accurately



Set up and solve an equation to work out the value of x.

$11(2x) + 3x + 2x = 56$

$\Rightarrow 16x = 56$

$\Rightarrow x = \frac{56}{16} = \frac{7}{2} = 3.5$

$x =$ 3.5 *(4 marks)*

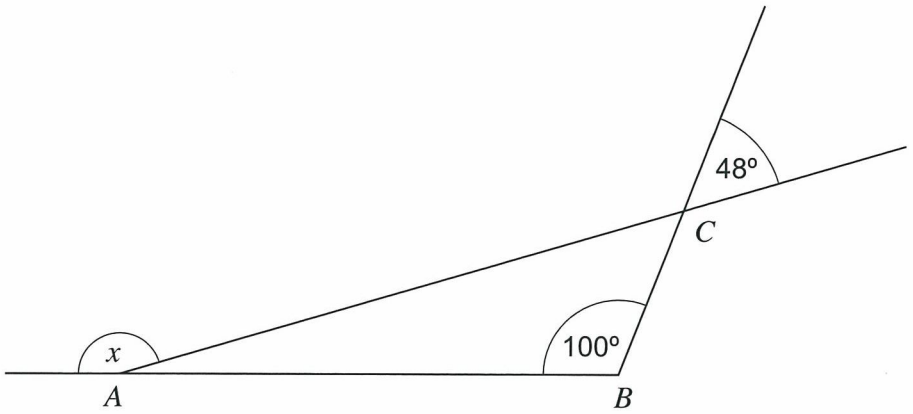
3 Work out the circumference of a circle, radius 4.2 cm.
Give your answer to 1 decimal place.

$C = 2\pi r = 2\pi(4.2) = 26.4$ cm (1 d.p.)

Answer 26.4 cm *(3 marks)*

4 The diagram shows a triangle ABC with sides extended.

Not drawn accurately



Work out the value of x.

$ACB = 48°$ —— Vertically opposite angles are equal.

$BAC = 180 - (100 + 48) = 180 - 148$

$= 32°$ —— Angles of a triangle add to 180°

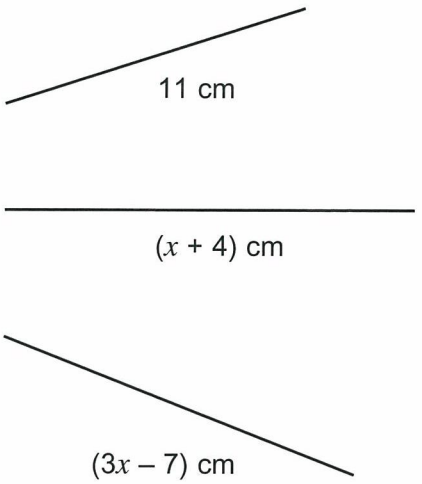
$x = 180 - 32 = 148°$ since angles across a straight line add to 180°

Answer 148 degrees *(3 marks)*

Turn over for the next question

10

5 The diagram shows three rods.



Not drawn accurately

Two of the rods are the same length.

Work out the **three** possible values for x.

$x + 4 = 11 \Rightarrow x = 11 - 4 = 7$

$3x - 7 = 11 \Rightarrow x = \frac{11+7}{3} = \frac{18}{3} = 6$

$3x - 7 = x + 4$

$\Rightarrow 2x - 7 = 4$

$\Rightarrow x = \frac{4+7}{2} = \frac{11}{2} = 5.5$

Answer 1 $x =$ 7

Answer 2 $x =$ 6

Answer 3 $x =$ 5.5 *(5 marks)*

***6** Here are two bottles of the same perfume.

Normal price £40

20% off

£55

Which is the better value?
You **must** show your working.

40 - 20% of 40 = 40 - 8 = £32
(or 0.8 x 40 = £32).

One way to compare the prices like-for-like is to scale-up the price of the small bottle in proportion with its volume until you have its price per 80ml.

$32 \times \frac{80}{50} = 32 \times \frac{8}{5} = \frac{256}{5} = £51.20$

∴ Small bottle is better value.

(6 marks)

11

7 (a) The scale on a map is 1 : 250 000

What is the actual distance represented by 1 centimetre?
Give your answer in kilometres.

1m = 100 cm and 1km = 1000m so

1km = 100,000 cm. $\frac{250,000}{100,000} = 2.5$ km

Answer 2.5 km *(3 marks)*

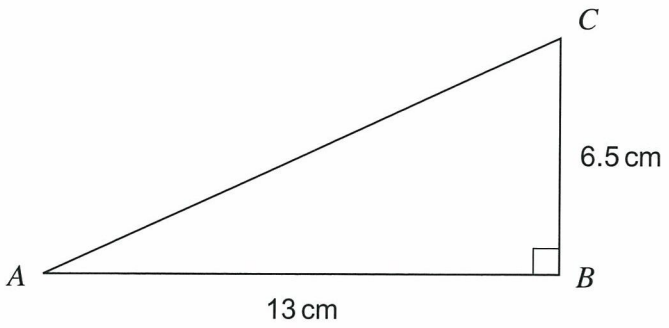
7 (b) The scale on a different map is 1 inch represents 4 miles.
A road on the map measures 6 inches to the nearest inch.

What is the shortest possible distance of the road?

Lower bound of distance of road is given by 5.5 × 4 = 22 miles.

Answer 22 miles *(3 marks)*

8 Work out the length AC.



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$\sqrt{13^2+6.5^2}$ = 14.5 cm (3 s.f.).

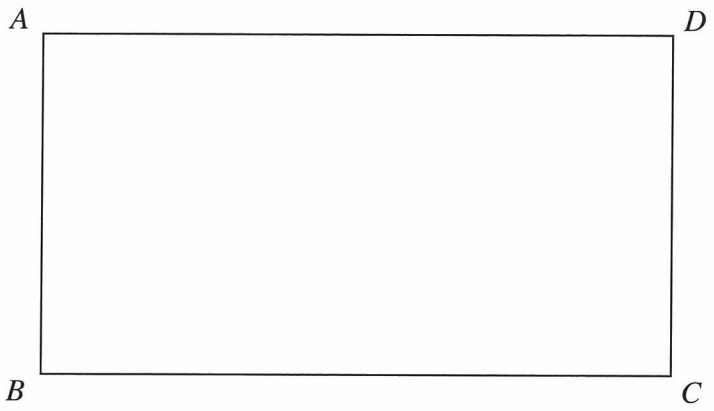
Answer 14.5 cm (3 s.f.) *(3 marks)*

Turn over for the next question

9

Turn over ▶

9 The diagram shows a rectangular sheet of paper $ABCD$.



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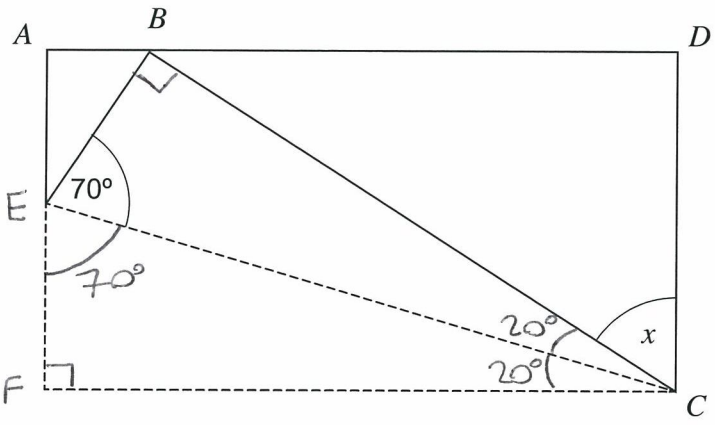
Corner B is folded to meet side AD as shown.



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Work out the angle marked x on the diagram.

BEFC is a kite which is made up of two congruent triangles BCE and EFC

FCE = ECB = 180 - (90 + 70) = 180 - 160 = 20° since angles of a triangle adel to 180°

$x = 90 - 40 = 50°$

Answer 50 degrees *(4 marks)*

10 Use trial and improvement to find a solution to $x^3 - 20x = 60$
Give your answer to 1 decimal place.

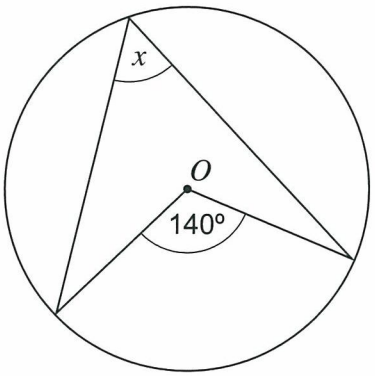
x	$x^3 - 20x$	Comment
5	25	Too small
6	96	too big
5·5	56·375	too small
5·6	63·616	too big
5·55	59·953875	too small
∴ 5·55 < x <	5·6 & so x =	5·6 (1 d.p.).

x = 5·6 (1 d.p.). *(4 marks)*

8

Turn over ▶

11 (a) The diagram shows a circle, centre O.



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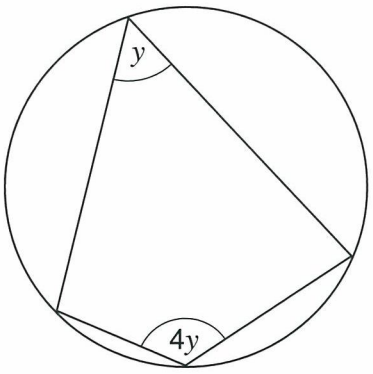
Work out the value of x.

$x = \frac{1}{2}(140) = 70°$ as per 'The Double Angle Theorem'

Answer 70 degrees *(1 mark)*

11 (b) The diagram shows a cyclic quadrilateral.



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Work out the value of y.

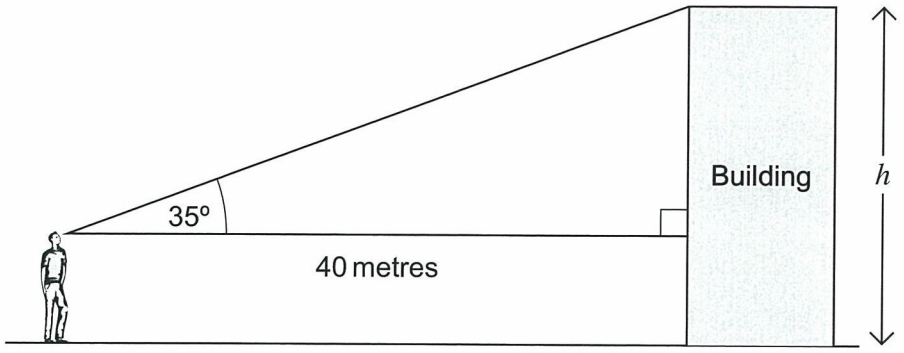
$4y + y = 180°$ since opposite angles of a cyclic quadrilateral add to 180°.

$\Rightarrow 5y = 180 \Rightarrow y = \frac{180}{5} = 36°$

Answer 36 degrees *(2 marks)*

12

Not drawn accurately



The man is 1.8 metres tall.

Work out the height of the building, marked h on the diagram.
Give your answer to a suitable degree of accuracy.

$h = 40\tan 35° + 1.8 = 29.8\text{m}$ (3s.f.).

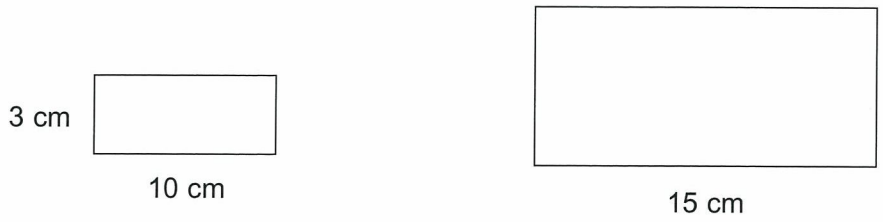
Answer 29.8 (3s.f.). metres *(5 marks)*

Turn over for the next question

8

13 Here are two similar rectangles.

Not drawn accurately



Work out the area of the larger rectangle.

$15 \times \left(3 \times \frac{3}{2}\right) = 15 \times 4.5 = 67.5\,cm^2$

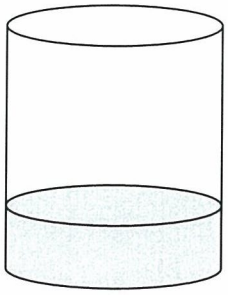
OR Area of larger = Area of smaller $\times \left(\frac{15}{10}\right)^2$

$\Rightarrow A_L = 30 \times \left(\frac{3}{2}\right)^2 = 30 \times \frac{9}{4}$

$= \frac{270}{4} = \frac{135}{2} = 67.5\,cm^2$

Answer 67.5 cm^2 *(5 marks)*

14 The cylindrical tank is one-quarter full of oil.



1 litre = 1000 cm^3

The radius of the base of the cylinder is 90 cm.
The height of the cylinder is 200 cm.

Work out the number of litres of oil in the tank.

Volume of cylinder $= \pi r^2 h = \pi(90^2)(200)$

$= 1{,}620{,}000\pi\ cm^3$ (or $1.62 \times 10^6 \pi\ cm^3$)

Volume of oil $= \frac{1}{4} \times 1{,}620{,}000\pi$

$= 405{,}000\pi\ cm^3$

$\frac{405{,}000\pi}{1000} = 405\pi$ litres

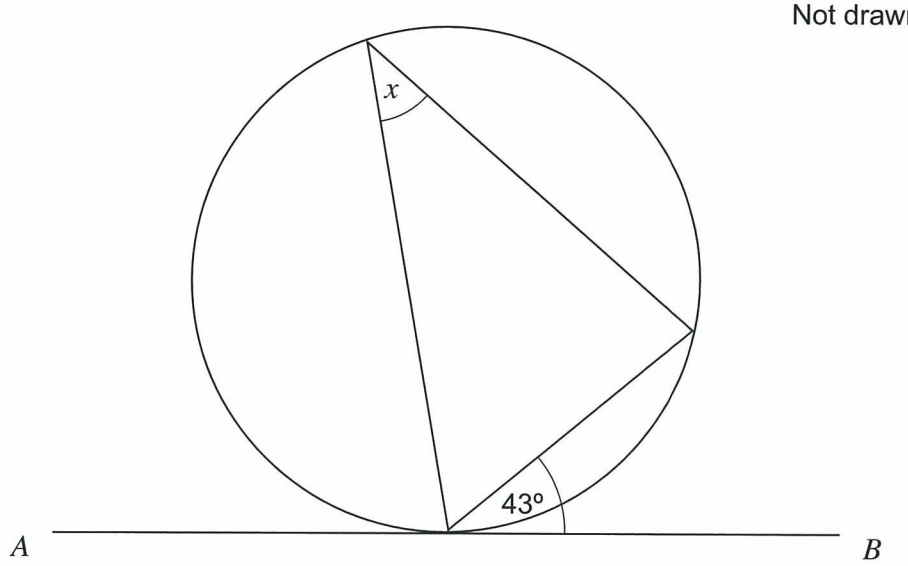
or 1272.35 litres (2d.p.).

Answer 1272.35 (2d.p.) litres *(4 marks)*

9

***15** AB is a tangent to the circle.

Not drawn accurately



Write down the value of x.
Give a reason for your answer.

Answer 43 degrees

Reason The angle in the opposite segment is equal.

(2 marks)

i.e. the angle between a chord and a tangent is equal to the angle subtended by the same chord at any other point on the circumference in the opposite segment.

16 Use the quadratic formula to solve

$$6x^2 + 5x - 3 = 0$$

Give your answers to 2 decimal places.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-5 \pm \sqrt{5^2 - 4(6)(-3)}}{2(6)}$

$= \dfrac{-5 \pm \sqrt{97}}{12}$

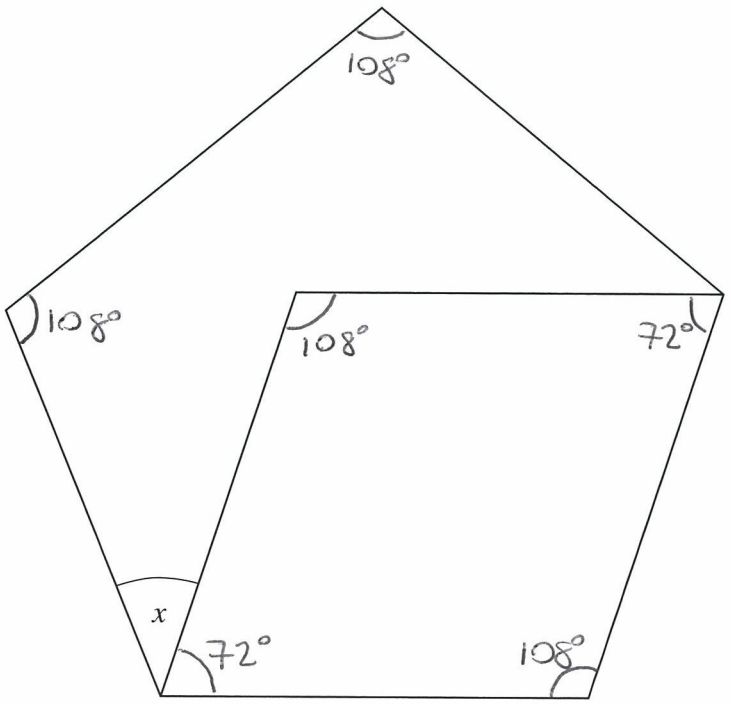
$= 0.40$ (2 d.p.) or -1.24 (2 d.p.).

Answer 0.40 and −1.24 *(3 marks)*

Turn over for the next question

5

17 The diagram shows a rhombus inside a regular pentagon.



Not drawn accurately

Work out the value of x.

For any regular n-sided polygon, exterior angle $e = \frac{360}{n}$ and $i = 180 - \frac{360}{n}$.

So the interior angles of a regular pentagon are given by $180 - \frac{360}{5} = 180 - 72 = 108°$

Also, angles of any quadrilateral add to 360° and the opposite angles of a rhombus are equal so

$x = 108 - \left(\frac{360 - 2(108)}{2}\right) = 108 - 72 = 36°$

Answer 36 degrees *(4 marks)*

18 (a) Here are four equations connecting y and x.
k is a constant.

$y = kx$ $\quad y = \frac{k}{x}$ $\quad y = kx^2$ $\quad y = \frac{k}{x^2}$

Match each equation to its statement.

y is **directly** proportional to x Equation $y = kx$

y is **directly** proportional to x^2 Equation $y = kx^2$

y is **inversely** proportional to x Equation $y = \frac{k}{x}$

y is **inversely** proportional to x^2 Equation $y = \frac{k}{x^2}$

(2 marks)

18 (b) y is **inversely** proportional to x.
When $x = 3$, $y = 8$

Work out the value of y when $x = 5$

$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x} \Rightarrow k = yx = 8(3) = 24$

So $y = \frac{24}{x}$

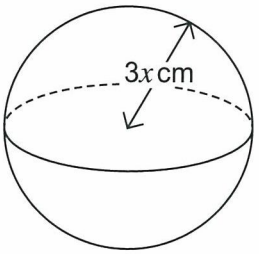
When $x = 5$, $y = \frac{24}{5} = 4\frac{4}{5}$ or 4.8

Answer $y = 4.8$ *(3 marks)*

9

Turn over ▶

19 (a) A sphere has radius $3x$ cm.



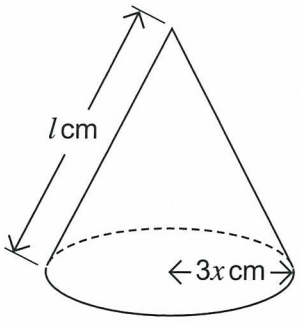
Write down an expression for the surface area of the sphere in terms of π and x.
Give your answer in its simplest form.

Surface area of sphere $= 4\pi r^2 = 4\pi(3x)^2$

$= 4\pi(9x^2) = 36\pi x^2$

Answer $36\pi x^2$ cm^2 *(2 marks)*

19 (b) A cone has base radius $3x$ cm and slant height l cm.



The curved surface area of the cone is equal to the surface area of the sphere.

Express l in terms of x.
Give your answer in its simplest form.

Curved surface area of cone $= \pi r l = 3\pi x l$

So $3\pi x l = 36\pi x^2$

$\Rightarrow l = \frac{36\pi x^2}{3\pi x} = 12x$

$\therefore l = 12x$

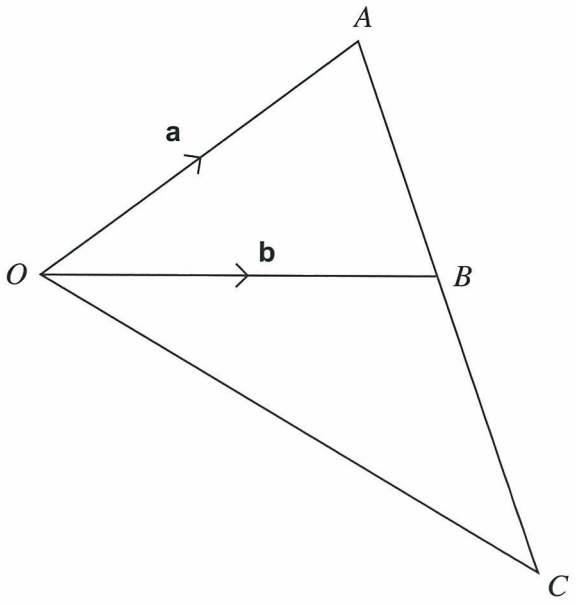
$l =$ $12x$ *(2 marks)*

Turn over for the next question

4

20 The diagram shows vectors $\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**

Not drawn accurately



20 (a) Write vector $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b$ or $b - a$

Answer $b - a$ *(1 mark)*

20 (b) The point B divides $\overrightarrow{AC}$ in the ratio 2 : 3

Work out vector $\overrightarrow{OC}$ in terms of **a** and **b**.

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$= b + \frac{3}{2}\overrightarrow{AB}$

$= b + \frac{3}{2}(b - a)$

$= b + \frac{3}{2}b - \frac{3}{2}a$

$= \frac{5}{2}b - \frac{3}{2}a$

$= \frac{1}{2}(5b - 3a)$

Answer $\frac{1}{2}(5b - 3a)$ *(3 marks)*

END OF QUESTIONS

4

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